Eur. Phys. J. D **20**, 281–292 (2002)
DOI: 10.1140/epjd/e2002-00131-2

THE EUROPEAN PHYSICAL JOURNAL D

Storage states in ultracold collective atoms

Kaige Wang[1,2,a] and Shiyao Zhu[2]

[1] CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, P.R. China
and
Department of Physics, Beijing Normal University, Beijing 100875, P.R. China
[2] Department of Physics, Hong Kong Baptist University, Hong Kong, P.R. China

Received 15 January 2002 and Received in final form 8 April 2002
Published online 19 July 2002 –

Abstract. We present a complete theoretical description of atomic storage states in the multimode framework by including spatial coherence in atomic collective operators and atomic storage states. We show that atomic storage states are Dicke states with the maximum cooperation number. In some limits, a set of multimode atomic storage states has been established in correspondence with multimode Fock states of the electromagnetic field. This gives better understanding of both the quantum and coherent information of optical field can be preserved and recovered in ultracold medium. In this treatment, we discuss in detail both the adiabatic and dynamic transfer of quantum information between the field and the ultracold medium.

PACS. 42.50.Gy Effects of atomic coherence on propagation, absorption, and amplification of light – 03.67.-a Quantum information

1 Introduction

In the interaction between optical fields and atoms, a photon can be absorbed by an atom and then the excited atom can re-emit a photon by either spontaneous or stimulated emission. In this process the atom stores the energy of the field and releases it back to the field. Recently, theoretical and experimental studies have shown that both the quantum state and coherent information of the field can be stored in an atomic medium [1–9]. A very recent experiment witnesses that a signal pulse of light can be stored in an ultracold collective of atoms for up to a millisecond [1].

The basic scheme for storage of light information is carried out by electromagnetically induced transparency (EIT) [10,11]. N three-level atoms with one upper level $|a\rangle$ and two lower levels $|b\rangle$ and $|c\rangle$ interact resonantly with both the signal and the control beams. The weak signal beam and the strong control beam drive the atomic transitions $|a\rangle - |b\rangle$ and $|a\rangle - |c\rangle$, respectively. Early investigations have shown that EIT permits the propagation of the light signal through an otherwise opaque atomic medium and that the group velocity of the signal pulse is greatly reduced [12–14]. In a recent experiment [1], when the control beam is turned off, the signal pulse is stopped and stored in atomic medium. This effect can be understood in terms of dark states. Dark states are combination states of the photon state and the atomic storage state. The bosonic quasiparticles in the dark state are called polaritons. When the strength of the control field is changed adiabatically, both the quantum state and the coherent information transfer between the signal field and the collective atoms [3,4].

Due to the fact that dark states are eigenstates of the EIT interaction Hamiltonian, the transfer of the quantum state between the field and the atomic ensemble can be quasi-stationary in the adiabatic limit. Furthermore, mapping and storage of quantum information of light in an atomic medium may occur in a dynamical process without forming dark states. For example, reference [7] has shown that in a general Raman interaction, with a large detuning to the intermediate level, the nonclassical features of the quantum field can be mapped onto the coherence of the lower atomic doublet, distributed over the atomic cloud. However, a very recent theoretical study has shown that, in the EIT model, an adiabatic change of the control field is not necessary, and even a fast switching of the control field can be used in the writing and reading quantum information of the signal field [8].

In all of these models, collective atoms play the role of quantum memory. The mechanism of storage of the quantum field in a medium is based on establishing atomic storage states (or atomic memory states) which record all the information of the field. It is not surprising that photons can be transferred to atomic excitations in transition interactions. Physically, the question is how collective atoms record the coherence of an electromagnetic field.

[a] e-mail: wangkg@bnu.edu.cn

In the early theoretical work [15], Dicke studied the coherence and cooperation effects in atomic ensemble. He defined collective atomic operators as the sum of all the individual atomic operators, retaining the angular momentum properties. The Dicke states are the eigenstates of the angular momentum operators. In Dicke's paper, he considered two cases: the gas volumes have dimensions either smaller or larger than the radiation wavelength. For the latter case, the spatial phase distribution of the field has been included into the collective atomic operators.

In the pioneering work contributed by Fleischhauer and Lukin [3–5, 9] the theory of atomic storage states basically consists of a single-mode description. In this paper, we present a complete description of atomic storage states in a multimode framework. A preliminary version of the briefly analysis described in this paper is given in reference [16]. Similarly, as in the second case of Dicke's work, we incorporate spatial coherence into the collective atomic lower and upper operators. We indicate that, in the multimode description, atomic collective operators can behave as multimode bosonic operators, under the conditions of low atomic excitation and appropriate radiation wavelength; which is much larger than the average interval of atoms and less than the propagation length of medium. Aside from the original definition of Dicke states, we introduce atomic storage states with explicit expressions by containing spatial coherence of the radiation field, and indicate that they are Dicke states with the maximum cooperation number. The significant advance is that, under the conditions shown above, a set of multimode atomic storage states is established in correspondence with multimode Fock states of the electromagnetic field. This gives a better understanding of how both quantum and coherent information, of electromagnetic fields, can be preserved in atomic media. A detailed theoretical description of multimode dark states in the EIT model is discussed. Furthermore, parallel to the "stationary polariton" in EIT, we show the "dynamic polariton" formed in coupled harmonic oscillators. This illustrates the mechanism for the dynamic quantum transfer between field and macroscopic matter.

2 Atomic collective operators with the bosonic commutation

We consider N ultracold collective atoms which are approximately stationary at their positions. At very low temperature close to the critical temperature for Bose-Einstein condensation [1], the average kinetic energy of atoms is greatly reduced. On the other hand, at low temperature, atoms are densely packed within a wavelength of optical field. The free path of an atom is much less than the wavelength, hence it confines the range of atomic motion. What the "still atoms" means is that, in the characteristic time of the system, the scale of motion for the centre-of-mass of the atoms is much less than the wavelength of the optical electromagnetic field involved. The two levels of atoms $|b\rangle$ and $|c\rangle$ interact with some optical field of wavevector k. We assume that N is a large number and the largest proportion of the population of the atoms is in level $|b\rangle$ throughout the system evolution, so that the completeness relation is given by

$$N = \sum_{j=1}^{N} (|b_j\rangle\langle b_j| + |c_j\rangle\langle c_j|) \simeq \sum_{j=1}^{N} |b_j\rangle\langle b_j|. \qquad (1)$$

It is not necessary that $|b\rangle$ is the ground state, for instance, in the case of EIT the level $|c\rangle$ can be lower or equal to $|b\rangle$. For the sake of convenience, we call $|b\rangle$ the "ground" state and $|c\rangle$ the "excited" state.

In the interaction between field and atomic medium, the spatial coherence of the field affects only the local atoms. In the approximation of "still atoms", the jth atom located at position z_j suffers a local field strength with a phase $\exp(ikz_j)$. For this reason, we define the lower and the upper operators of the collective atoms as

$$\sigma_k = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |b_j\rangle\langle c_j| \exp(-\mathrm{i}kz_j), \qquad (2a)$$

$$\sigma_k^\dagger = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |c_j\rangle\langle b_j| \exp(\mathrm{i}kz_j), \qquad (2b)$$

where k is the wavevector of the optical electromagnetic field interacting with the transition $|b\rangle - |c\rangle$. We notice that this kind of collective atomic operator, containing spatial coherence, was first introduced by Dicke, who investigated the super-radiate effect in collective atoms, in the case of medium dimensions larger than radiation wavelength [15]. In order to avoid the difficulties of the occurrence of the center-of-mass motion of atoms which may destroy the coherence, in Dicke's paper, he assumed the molecules are so massive that their center-of-mass coordinates will be then treated as time-independent parameter in equation. Now, this assumption can be implemented in the development of the ultracold technique.

For the purpose of controllable storage, the atomic transition $|b\rangle - |c\rangle$ is usually a multi-photon process including signal and control photons. Equation (2a) should be replaced by

$$\sigma_k = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |b_j\rangle\langle c_j| \exp[-\mathrm{i}(k - k_\mathrm{c})z_j], \qquad (3a)$$

$$\sigma_k = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |b_j\rangle\langle c_j| \exp[-\mathrm{i}(k + k_\mathrm{c})z_j], \qquad (3b)$$

where k and k_c are respectively the wavevectors for the signal field and the control field. Equation (3a) describes a Raman transition, whereas equation (3b) describes a two-photon cascade transition.

The commutation relations for these atomic operators are written as

$$[\sigma_k, \sigma_{k'}] = [\sigma_k^\dagger, \sigma_{k'}^\dagger] = 0, \tag{4a}$$

$$[\sigma_k, \sigma_{k'}^\dagger] = (1/N)\sum_{j=1}^{N}(|b_j\rangle\langle b_j| - |c_j\rangle\langle c_j|)\exp[-\mathrm{i}(k-k')z_j]. \tag{4b}$$

The exact commutation of equation (4b) for the same mode is readily obtained

$$[\sigma_k, \sigma_k^\dagger] = (1/N)\sum_{j=1}^{N}(|b_j\rangle\langle b_j| - |c_j\rangle\langle c_j|). \tag{5}$$

If N is a very large number and most of the atomic population rests in level $|b\rangle$ throughout evolution, by applying equation (1), then equation (4b) is approximately reduced to

$$[\sigma_k, \sigma_{k'}^\dagger] \simeq (1/N)\sum_{j=1}^{N}\exp[-\mathrm{i}(k-k')z_j]. \tag{6}$$

Assuming that the atoms are in a string and the average interval of the adjacent atoms is d, which is much less than the optical wavelength, *i.e.* $kd \ll 1$, one obtains

$$\sum_{j=1}^{N}\exp[\mathrm{i}kz_j] = \sum_{j=1}^{N}\exp[\mathrm{i}k(j-1)d] = \frac{1-\exp[\mathrm{i}kNd]}{1-\exp[\mathrm{i}kd]} \approx N\frac{\exp[\mathrm{i}kL]-1}{\mathrm{i}kL}, \tag{7}$$

where $L = Nd$ is the length of the atomic medium. However, this result is also true for a volume of atomic gas which is considered as a continuous medium

$$\sum_{j=1}^{N}\exp[\mathrm{i}kz_j] = \int_0^L \frac{N}{L}\exp[\mathrm{i}kz]\mathrm{d}z = N\frac{\exp[\mathrm{i}kL]-1}{\mathrm{i}kL}\cdot \tag{8}$$

In the case that the length of the atomic medium is much larger than the optical wavelength, we obtain

$$\frac{1}{N}\sum_{j=1}^{N}\exp[\mathrm{i}kz_j] = \begin{cases} 1 & (k=0), \\ 0 & (kL \gg 1). \end{cases} \tag{9}$$

By applying the above result to equation (6), one obtains the bosonic commutation relation for the collective atomic operators

$$[\sigma_k, \sigma_{k'}^\dagger] \simeq \delta_{kk'}, \tag{10}$$

where we should assume $(k-k')L \gg 1$, or, equivalently, $\lambda - \lambda' \gg \lambda^2/(2\pi L)$. For the parameters used in the experiment [1], $L = 339$ μm and $\lambda = 589.6$ nm, so that $\lambda^2/(2\pi L) \approx 0.163$ nm, equation (10) is a good approximation for distinguishable modes.

We summarize the conditions for the collective atomic operators satisfying the multimode bosonic commutation as

$$N \gg n, \tag{11a}$$

$$\lambda, L \gg d, \tag{11b}$$

$$\Delta\lambda/\lambda \gg \lambda/L, \tag{11c}$$

where n is the number of atomic excitations and $\Delta\lambda$ is the mode interval. The low excitation limit (11a) has already been shown in the previous paper [3]. The other two conditions imposed on the radiation wavelength, equations (11b, 11c), assure the atomic ensemble containing and distinguishing the coherence, respectively. We will see in the next section that the atomic collective operators behave similarly to the creation and annihilation operators of the electromagnetic field.

3 Single-mode atomic storage states

The "ground-level" state of the atoms can be compared with the "vacuum" state, symbolized in reference [3] as

$$|C^0\rangle \equiv |b_1 b_2 \cdots b_N\rangle. \tag{12}$$

When the single-mode creation operators of the collective atoms are applied to the "vacuum" state, one obtains

$$\begin{aligned}(\sigma_k^\dagger)^n|C^0\rangle &= \frac{1}{\sqrt{N^n}}\left(\sum_{j=1}^{N}|c_j\rangle\langle b_j|\exp(\mathrm{i}kz_j)\right)^n|b_1 b_2\cdots b_N\rangle \\ &= \frac{1}{\sqrt{N^n}}\sum_{\{i_n\}}{}'|c_{i_1}\cdots c_{i_n}\rangle\langle b_{i_1}\cdots b_{i_n}|b_1 b_2\cdots b_N\rangle \\ &\quad\times\exp[\mathrm{i}k(z_{i_1}+\cdots+z_{i_n})] \\ &= \frac{1}{\sqrt{N^n}}\sum_{\{i_n\}}{}'|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \\ &\quad\times\exp[\mathrm{i}k(z_{i_1}+\cdots+z_{i_n})]\end{aligned} \tag{13}$$

where $\sum'_{\{i_n\}}$ designates that, in the summation, any two indices cannot be equal, because $(|c_j\rangle\langle b_j|)^2|b_j\rangle = 0$. We note that some states in the summation of equation (13), for which the sequence in the index set $\{i_n\}$ is exchanged, are the same and should be put together. For example, $(i_1 = 1, i_2 = 2, i_3, ..., i_n)$ and $(i_1 = 2, i_2 = 1, i_3, ..., i_n)$ represent the same state. For an ensemble $\{i_n\}$ of n elements, there are $n!$ permutations which form the same state. By eliminating these repeated terms in the summation, equation (13) can be replaced by

$$(\sigma_k^\dagger)^n|C^0\rangle = \frac{n!}{\sqrt{N^n}}\sum_{\{i_n\}}{}''|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \times\exp[\mathrm{i}k(z_{i_1}+\cdots+z_{i_n})], \tag{14}$$

where $\sum''_{\{i_n\}}$ is defined as

$$\sum_{\{i_n\}}{}'' \equiv \underbrace{\sum_{i_1=1}^{N-n+1}\sum_{i_2=2}^{N-n+2}\cdots\sum_{i_{n-1}=n-1}^{N-1}\sum_{i_n=n}^{N}}_{\{i_1<i_2<\cdots<i_{n-1}<i_n\}}. \quad (15)$$

The summation of equation (14) includes $\binom{N}{n} = N(N-1)\cdots(N-n+1)/n!$ terms. Now, we define a normalized atomic storage state

$$|C_k^n\rangle = \sqrt{\frac{n!}{N(N-1)\cdots(N-n+1)}} \times \sum_{\{i_n\}}{}''|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \exp[\mathrm{i}k(z_{i_1}+\cdots+z_{i_n})]. \quad (16)$$

Obviously, the atomic storage states with a different number of excitations are orthogonal to each other

$$\langle C_k^n|C_k^m\rangle = \delta_{nm}. \quad (17)$$

In this definition, the superposition state of N collective atoms includes any possible combination of n atoms being in the level $|c\rangle$, while the corresponding spatial coherence is recorded in the phase of the wavefunction. Physically, it means that n photons can be stored by any combination of excited n atoms with an equal possibility, in correspondence with the nonlocality for photons. However, the coherent information of the field has been retained in the probability amplitudes. Note that the atomic storage state, with a definite wavevector k, is independent of position z, which disappears in the summation.

By using definition (16), equation (14) becomes

$$(\sigma_k^\dagger)^n|C^0\rangle = \sqrt{\frac{N(N-1)\cdots(N-n+1)}{N^n}}\sqrt{n!}|C_k^n\rangle \cdot \quad (18)$$

It is easy to check that

$$\sigma_k^\dagger|C_k^n\rangle = \sqrt{1-\frac{n}{N}}\sqrt{n+1}|C_k^{n+1}\rangle \cdot \quad (19)$$

The above two equations are exact. However, in the limit $N \gg n$, the corresponding approximate expressions are

$$(\sigma_k^\dagger)^n|C^0\rangle \simeq \sqrt{n!}|C_k^n\rangle, \quad (20)$$

and

$$\sigma_k^\dagger|C_k^n\rangle \simeq \sqrt{n+1}|C_k^{n+1}\rangle \cdot \quad (21)$$

The annihilation operator is applied to the "vacuum" state

$$\sigma_k|C^0\rangle = 0. \quad (22)$$

In Appendix A, the general formula for the annihilation operator is proved as

$$\sigma_k|C_k^n\rangle = \sqrt{1-\frac{n-1}{N}}\sqrt{n}|C_k^{n-1}\rangle \simeq \sqrt{n}|C_k^{n-1}\rangle \cdot \quad (23)$$

Equations (19, 23) give immediately

$$\sigma_k^\dagger\sigma_k|C_k^n\rangle = (1-\frac{n-1}{N})n|C_k^n\rangle \simeq n|C_k^n\rangle, \quad (24a)$$

$$\sigma_k\sigma_k^\dagger|C_k^n\rangle = (1-\frac{n}{N})(n+1)|C_k^n\rangle \simeq (n+1)|C_k^n\rangle. \quad (24b)$$

The approximations in equations (23, 24) are valid in the limit $N \gg n$. Equation (24) verifies again the bosonic commutation in this limit. If one admits both the bosonic commutation (10) and equation (21), by using the commutation

$$[\sigma_k, (\sigma_k^\dagger)^n] \simeq n(\sigma_k^\dagger)^{n-1}, \quad (25)$$

it can also obtain

$$\sigma_k|C_k^n\rangle \simeq \sqrt{n}|C_k^{n-1}\rangle \cdot \quad (26)$$

The atomic storage states are also the eigenstates of the population operators

$$\sum_{j=1}^{N}(|b_j\rangle\langle b_j|C_k^n\rangle = (N-n)|C_k^n\rangle, \quad (27a)$$

$$\sum_{j=1}^{N}(|c_j\rangle\langle c_j|C_k^n\rangle = n|C_k^n\rangle \cdot \quad (27b)$$

(see Appendix A)

According to Dicke's definition (Eq. (47) in Ref. [15]), the total angular momentum operators of the atomic ensemble can be described as

$$R_{k1} = (\sqrt{N}/2)(\sigma_k^\dagger + \sigma_k), \quad (28a)$$

$$R_{k2} = (-\mathrm{i}\sqrt{N}/2)(\sigma_k^\dagger - \sigma_k), \quad (28b)$$

$$R_3 = (N/2)\left(\sigma_k^\dagger\sigma_k - \sigma_k\sigma_k^\dagger\right), \quad (28c)$$

$$R^2 = R_{k1}^2 + R_{k2}^2 + R_3^2 = (N/2)\left(\sigma_k^\dagger\sigma_k + \sigma_k\sigma_k^\dagger\right) + (N^2/4)\left(\sigma_k^\dagger\sigma_k - \sigma_k\sigma_k^\dagger\right)^2. \quad (28d)$$

Using the exact relations of equations (19, 23), one obtains

$$R_3|C_k^n\rangle = \frac{1}{2}(2n-N)|C_k^n\rangle, \quad (29a)$$

$$R^2|C_k^n\rangle = \frac{1}{2}N(\frac{1}{2}N+1)|C_k^n\rangle \cdot \quad (29b)$$

Therefore, the atomic storage state defined in equation (16) is the right Dicke state with the maximum cooperation number $r = N/2$. The discussion in this section exploits a new feature of Dicke state. The Dicke states with the maximum cooperation number play the role of number states in front of the collective lower and upper atomic operators.

4 Multimode atomic storage states

The multimode case is concerned with how the information of the multimode photons is distributed in the local atomic excitations $|c_j\rangle$. To see it, we firstly consider a simple case — the multimode single-excitation atomic storage state; that is, each mode contains only one excitation. We apply the multimode creation operators to the "vacuum" state

$$\begin{aligned}\sigma^\dagger_{k_1}\cdots\sigma^\dagger_{k_n}|C^0\rangle &= \frac{1}{\sqrt{N^n}}\sum_{i_1=1}^{N}|c_{i_1}\rangle\langle b_{i_1}|\exp(\mathrm{i}k_1z_{i_1})\cdots\\ &\times\sum_{i_n=1}^{N}|c_{i_n}\rangle\langle b_{i_n}|\exp(\mathrm{i}k_nz_{i_n})|b_1b_2\cdots b_N\rangle\\ &= \frac{1}{\sqrt{N^n}}\sum_{\{i_n\}}{}'|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle\\ &\times\exp[\mathrm{i}(k_1z_{i_1}+\cdots+k_nz_{i_n})]. \qquad (30)\end{aligned}$$

This equation is apparently different to equation (13) by the phase factors. Indeed, the exchanges of the indices in the summation contribute to the same atomic state, but, with different phase distributions. For example, $(i_1=1, i_2=2, i_3, ..., i_n)$ and $(i_1=2, i_2=1, i_3, ..., i_n)$ display the same state $|c_1c_2b_3\cdots c_{i_3}\cdots c_{i_n}\cdots b_N\rangle$, but with the phase factors $\exp[\mathrm{i}(k_1z_1+k_2z_2+k_3z_{i_3}+\cdots+k_nz_{i_n})]$ and $\exp[\mathrm{i}(k_1z_2+k_2z_1+k_3z_{i_3}+\cdots+k_nz_{i_n})]$, respectively. Mathematically, for a given atomic collective state $|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle$, it allocates $n!$ phase factors due to $n!$ permutations for n elements. This means that an atom in the level $|c_j\rangle$, located at position z_j, records the information of all the modes. Because the field is global, each atom in the medium experiences the field coherence of all the modes, and, *vice versa*, the field of each mode affects all the excited atoms.

To simplify the sign, we define

$$\{k_n\}\cdot\{z_{i_n}\}_l \equiv (k_1z_{i_1}+\cdots+k_nz_{i_n})_l, \qquad (31)$$

where $\{z_{i_n}\}_l$ stands for the lth sequence of all the $n!$ permutations for n elements. Accordingly, equation (30) can be written as

$$\begin{aligned}\sigma^\dagger_{k_1}\cdots\sigma^\dagger_{k_n}|C^0\rangle &= \frac{1}{\sqrt{N^n}}\sum_{\{i_n\}}{}''|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle\\ &\times\sum_{l=1}^{n!}\exp[\mathrm{i}\{k_n\}\cdot\{z_{i_n}\}_l], \qquad (32)\end{aligned}$$

where $\sum''_{\{i_n\}}$ has already been defined in equation (15). In comparison with the single mode case, shown in equation (14), n-excitations in equation (32) share the phases of n modes. We define a multimode single-excitation atomic storage state as

$$\begin{aligned}|C^1_{k_1}\cdots C^1_{k_n}\rangle &\equiv \frac{1}{\sqrt{N(N-1)\cdots(N-n+1)}}\\ &\times\sum_{\{i_n\}}{}''|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle\sum_{l=1}^{n!}\exp[\mathrm{i}\{k_n\}\cdot\{z_{i_n}\}_l], \qquad (33)\end{aligned}$$

which has been normalized, as shown in Appendix B. With the combination of equations (32, 33), we obtain

$$\begin{aligned}\sigma^\dagger_{k_1}\cdots\sigma^\dagger_{k_n}|C^0\rangle &= \sqrt{\frac{N(N-1)\cdots(N-n+1)}{N^n}}|C^1_{k_1}\cdots C^1_{k_n}\rangle\\ &\simeq|C^1_{k_1}\cdots C^1_{k_n}\rangle\,. \qquad (34)\end{aligned}$$

Now, we discuss the general case of multimode atomic storage states. s modes containing total n excitations can be generated by

$$\begin{aligned}(\sigma^\dagger_{k_1})^{m_1}\cdots(\sigma^\dagger_{k_\mathrm{s}})^{m_\mathrm{s}}|C^0\rangle &= \\ \frac{1}{\sqrt{N^n}}&\left(\sum_{j=1}^{N}|c_j\rangle\langle b_j|\exp(\mathrm{i}k_1z_j)\right)^{m_1}\cdots\\ \times&\left(\sum_{j=1}^{N}|c_j\rangle\langle b_j|\exp(\mathrm{i}k_\mathrm{s}z_j)\right)^{m_\mathrm{s}}|b_1b_2\cdots b_N\rangle \qquad (35)\end{aligned}$$

where $m_1+\cdots+m_\mathrm{s}=n$. By defining an index set as

$$\{i_n\}\equiv(i_1,...,i_{m_1},i_{m_1+1},...,i_{m_2},...,i_{n-m_\mathrm{s}+1},...,i_n), \qquad (36)$$

equation (35) can be written as

$$\begin{aligned}(\sigma^\dagger_{k_1})^{m_1}\cdots(\sigma^\dagger_{k_\mathrm{s}})^{m_\mathrm{s}}|C^0\rangle &= \\ \frac{1}{\sqrt{N^n}}&\sum_{\{i_n\}}{}'|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle\\ &\times\exp[\mathrm{i}k_1(z_{i_1}+\cdots+z_{i_{m_1}})+\cdots\\ &+\mathrm{i}k_\mathrm{s}(z_{i_{n-m_\mathrm{s}+1}}+\cdots+z_{i_n})]. \qquad (37)\end{aligned}$$

Similarly, as in the previous cases, any particular atomic collective state $|b_1\cdots c_{j_1}\cdots c_{j_n}\cdots b_N\rangle$ is related to $n!$ terms in the summation throughout all indices. But, among these $n!$ terms, the phase factor of each term will repeatedly appear $m_1!\cdots m_\mathrm{s}!$ times because exchanges of indices within a mode cause no difference. As a result, the remaining non-repeated phase factors terms are $n!/(m_1!\cdots m_\mathrm{s}!)$. We define again

$$\begin{aligned}\{k_\mathrm{s}^{(m_\mathrm{s})}\}\cdot\{z_{i_n}\}_l &\equiv (k_1(z_{i_1}+\cdots+z_{i_{m_1}})\\ &+\cdots+k_\mathrm{s}(z_{i_{n-m_\mathrm{s}+1}}+\cdots+z_{i_n}))_l \qquad (38)\end{aligned}$$

as one of these combinations with index l. Equation (37) is written as

$$\begin{aligned}(\sigma^\dagger_{k_1})^{m_1}\cdots(\sigma^\dagger_{k_\mathrm{s}})^{m_\mathrm{s}}|C^0\rangle &= \\ \frac{m_1!\cdots m_\mathrm{s}!}{\sqrt{N^n}}&\sum_{\{i_n\}}{}''|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle\\ &\times\sum_{l=1}^{n!/(m_1!\cdots m_\mathrm{s}!)}\exp[\mathrm{i}\{k_\mathrm{s}^{(m_\mathrm{s})}\}\cdot\{z_{i_n}\}_l]. \qquad (39)\end{aligned}$$

A general multimode atomic storage state can be defined as

$$|C_{k_1}^{m_1}\cdots C_{k_s}^{m_s}\rangle \equiv \sqrt{\frac{m_1!\cdots m_s!}{N(N-1)\cdots(N-n+1)}} \times \sum_{\{i_n\}}{}'' |b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \times \sum_{l=1}^{n!/(m_1!\cdots m_s!)} \exp[\mathrm{i}\{k_s^{(m_s)}\}\cdot\{z_{i_n}\}_l], \quad (40)$$

which has been normalized (see Appendix B). Finally, equation (39) becomes

$$(\sigma_{k_1}^\dagger)^{m_1}\cdots(\sigma_{k_s}^\dagger)^{m_s}|C^0\rangle = \sqrt{\frac{N(N-1)\cdots(N-n+1)}{N^n}} \times \sqrt{m_1!\cdots m_s!}|C_{k_1}^{m_1}\cdots C_{k_s}^{m_s}\rangle \simeq \sqrt{m_1!\cdots m_s!}|C_{k_1}^{m_1}\cdots C_{k_s}^{m_s}\rangle, \quad (41)$$

where the approximation is valid in the limit $N \gg n$. In this limit, the creation of an excitation of mode l for a multimode storage state is written as

$$\sigma_{k_l}^\dagger|C_{k_1}^{m_1}\cdots C_{k_l}^{m_l}\cdots C_{k_s}^{m_s}\rangle \simeq \sqrt{m_l+1}|C_{k_1}^{m_1}\cdots C_{k_l}^{m_l+1}\cdots C_{k_s}^{m_s}\rangle\cdot \quad (42)$$

Similarly, as in the single mode case, by considering the bosonic commutation (10) and equation (41), the annihilation of an excitation of mode l is written as

$$\sigma_{k_l}|C_{k_1}^{m_1}\cdots C_{k_l}^{m_l}\cdots C_{k_s}^{m_s}\rangle \simeq \sqrt{m_l}|C_{k_1}^{m_1}\cdots C_{k_l}^{m_l-1}\cdots C_{k_s}^{m_s}\rangle\cdot \quad (43)$$

Equations (18, 34, 41) imply that the state $|C^0\rangle$ defined in equation (12) represents a vacuum state not only for a single mode but also for a multimode. Physically, it shows that the ultracold collective atoms are able to store a multimode field. As a result, we may derive

$$\sigma_{k_2}^\dagger|C_{k_1}^{m_1}\rangle \simeq |C_{k_2}C_{k_1}^{m_1}\rangle, \quad (44)$$

where $|C_{k_1}^{m_1}\rangle$ can be understood as either a single mode state or a multimode state with the vacuum for those modes other than mode k_1.

In the above theoretical description, we have shown that the atomic storage states are a duplicate of the Fock states of the electromagnetic field. The explicit expressions of atomic storage states equations (16, 40) display duality of particle and coherence. The excitations may appear everywhere with an equal probability in the medium, corresponding with the nonlocality for photons. However, each excitation takes the local phase factors of the single-mode, or multimode, fields as the quantum probability amplitude, therefore recording the coherence. Consequently, it may establish the correspondence of two quantum system, the field and the atomic ensemble. This provides the basis for complete storage of quantum information of the bosonic field in an atomic medium.

5 Dark states in EIT

In the EIT configuration, the weak signal field interacting resonantly with the atomic transition $|a\rangle-|b\rangle$ is described by the field operator

$$E_s(z,t) = (1/2)\mathcal{E}_0 a(z)\exp[\mathrm{i}(k_s z-\omega_s t)] + \text{h.c.} = (1/2)\mathcal{E}_0\sum_q a(q)\exp[\mathrm{i}qz]\exp[\mathrm{i}(k_s z-\omega_s t)] + \text{h.c.}, \quad (45)$$

where $\mathcal{E}_0$ is the field amplitude per photon and $\omega_s = ck_s$. c is the speed of light in a vacuum. The strong control field, resonantly driving the atomic transition $|a\rangle-|c\rangle$ is assumed as classical

$$E_c(z,t) = \frac{\hbar\Omega}{2\wp_{ac}}\exp[\mathrm{i}(k_c z-\omega_c t)] + \text{c.c.}, \quad (46)$$

where $\wp_{ac}$ is the dipole moment of the transition $|a\rangle-|c\rangle$ and $\omega_c = ck_c$. In the interaction picture, the interaction Hamiltonian is described as

$$H_I = \hbar\sum_q \omega_q a^\dagger(q)a(q) - \frac{\hbar}{2}\sum_{j=1}^N\left\{g\sum_q a(q)|a_j\rangle\langle b_j|\exp[\mathrm{i}(k_s+q)z_j] + \Omega|a_j\rangle\langle c_j|\exp[\mathrm{i}k_c z_j] + \text{h.c.}\right\}, \quad (47)$$

where $\omega_q = cq$ is the detuning of mode q with respect to the resonant frequency ω_s of the signal field. By defining the atomic collective operators

$$\rho_{ab}(q) = \frac{1}{N}\sum_{j=1}^N |a_j\rangle\langle b_j|\exp[\mathrm{i}(k_s+q)z_j], \quad (48a)$$

$$\rho_{ac}(q) = \frac{1}{N}\sum_{j=1}^N |a_j\rangle\langle c_j|\exp[\mathrm{i}(k_c+q)z_j], \quad (48b)$$

the Hamiltonian (47) is written as

$$H_I = \hbar\sum_q \omega_q a^\dagger(q)a(q) - \frac{\hbar}{2}\left\{gN\sum_q a(q)\rho_{ab}(q) + \Omega N\rho_{ac}(0) + \text{h.c.}\right\}\cdot \quad (49)$$

The new quantum field operator defined in reference [3] is written as

$$\psi_q = \cos\theta a_q - \sin\theta\sigma_q, \quad (50)$$

where

$$\cos\theta = \Omega/\sqrt{\Omega^2+g^2N}, \quad \sin\theta = g\sqrt{N}/\sqrt{\Omega^2+g^2N}. \quad (51)$$

The transition $|b\rangle - |c\rangle$ concerns both the absorption of a signal photon and the emission of a driving photon. Replacing k by $(k_\mathrm{s}+q)-k_\mathrm{c}$ in equation (2), one obtains the annihilation operator σ_q of the collective atoms in EIT

$$\sigma_q = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |b_j\rangle\langle c_j| \exp[-\mathrm{i}(k_\mathrm{s}+q-k_\mathrm{c})z_j]. \quad (52)$$

Correspondingly, k should also be replaced by $k_\mathrm{s}+q-k_\mathrm{c}$ in the atomic storage states. ψ_q satisfies the bosonian commutation relation as long as σ_q does

$$[\psi_q, \psi_{q'}^\dagger] = \cos^2\theta[a_q, a_{q'}^\dagger] + \sin^2\theta[\sigma_q, \sigma_{q'}^\dagger] \simeq \delta_{qq'}. \quad (53)$$

It has been shown in equation (51), that the parameter θ is related to the strength of the control field. In the strong and weak limits of the control field, ψ_q tends to a_q and σ_q, respectively.

According to reference [3], the dark state is defined as

$$|D_q^n\rangle = \frac{1}{\sqrt{n!}}(\psi_q^\dagger)^n|0\rangle|C^0\rangle, \quad (54)$$

where $|0\rangle$ is the vacuum state of the signal field. The lowest dark state is designated as $|D^0\rangle \equiv |0\rangle|C^0\rangle$. The quasiparticle in the dark state is called a polariton [3].

Using equation (18), one obtains the exact expression of the dark state

$$\begin{aligned}|D_q^n\rangle &= \sum_{m=0}^{n}(-1)^m\sqrt{\frac{n(n-1)\cdots(n-m+1)}{m!}} \\ &\quad\times\sqrt{\frac{N(N-1)\cdots(N-m+1)}{N^m}} \\ &\quad\times\cos^{n-m}\theta\sin^m\theta|n-m\rangle|C_q^m\rangle \\ &= \sum_{m=0}^{n}\sqrt{\frac{n(n-1)\cdots(n-m+1)}{m!}} \\ &\quad\times\sqrt{\frac{N(N-1)\cdots(N-m+1)}{N^m}} \\ &\quad\times\frac{\Omega^{n-m}(-g\sqrt{N})^m}{(\Omega^2+g^2N)^{n/2}}|n-m\rangle|C_q^m\rangle\cdot \end{aligned} \quad (55)$$

The dark states described above are orthogonal to each other since they have different quasiparticle numbers, but are not normalized. Under the condition $N \gg n$, the dark state can be approximately written as

$$\begin{aligned}|D_q^n\rangle &\simeq \sum_{m=0}^{n}(-1)^m\sqrt{\frac{n(n-1)\cdots(n-m+1)}{m!}} \\ &\quad\times\cos^{n-m}\theta\sin^m\theta|n-m\rangle|C_q^m\rangle \\ &= \sum_{m=0}^{n}\sqrt{\frac{n(n-1)\cdots(n-m+1)}{m!}} \\ &\quad\times\frac{\Omega^{n-m}(-g\sqrt{N})^m}{(\Omega^2+g^2N)^{n/2}}|n-m\rangle|C_q^m\rangle\cdot \end{aligned} \quad (56)$$

The above expression of the dark state satisfies the normalized orthogonal relation

$$\langle D_q^n|D_q^m\rangle = \delta_{nm}. \quad (57)$$

Equation (56) shows that when the parameter θ is taken to be 0 and $\pi/2$, the summation in the dark states reduces to only the first and the last term

$$|D_q^n\rangle = |n\rangle|C^0\rangle \qquad \text{for} \quad \theta = 0, \quad (58a)$$
$$|D_q^n\rangle = (-1)^n|0\rangle|C_q^n\rangle \qquad \text{for} \quad \theta = \pi/2, \quad (58b)$$

respectively. Therefore, by varying θ adiabatically, the quasi-particles can be transferred between the photon state and the atomic storage state.

According to definition (54), one can obtain the exact expression

$$\psi_q^\dagger|D_q^n\rangle = \frac{1}{\sqrt{n!}}(\psi_q^\dagger)^{n+1}|0\rangle|C^0\rangle = \sqrt{n+1}|D_q^{n+1}\rangle\cdot \quad (59)$$

It is easy to check

$$\psi_q|D^0\rangle = 0. \quad (60)$$

As the same for the operator σ_q, with the help of the bosonic commutation relation (53), one obtains for the dark state (56)

$$\psi_q|D_q^n\rangle \simeq \sqrt{n}|D_q^{n-1}\rangle\cdot \quad (61)$$

Moreover, the multimode dark state can be generated by

$$|D_{q_1}^{n_1}\cdots D_{q_\mathrm{s}}^{n_\mathrm{s}}\rangle = \frac{1}{\sqrt{n_1!\cdots n_\mathrm{s}!}}(\psi_{q_1}^\dagger)^{n_1}\cdots(\psi_{q_\mathrm{s}}^\dagger)^{n_\mathrm{s}}|0\rangle|C^0\rangle. \quad (62)$$

They can be treated just like the multimode photon number states.

In Appendix C, we have proven that, at the exact resonance, both the exact expression (55) and the approximate expression (56) of the dark states are the eigenstates of the interaction Hamiltonian (49) with a null eigenvalue. A pulse of monochromatic light has a narrow bandwidth, and the detuning ω_q from the carrier frequency ω_s is small. If we omit the first term in the Hamiltonian (49), the multimode dark states consisting of the pulse are the eigenstates of the interaction Hamiltonian.

Assume that, at the initial time, a signal pulse is at a multimode state

$$\sum_{\{q_\mathrm{s}\}}\alpha(q_1,\cdots,q_\mathrm{s})|n_1\cdots n_\mathrm{s}\rangle, \quad (63)$$

while the cold collective atoms are, approximately, in the ground state $|C^0\rangle$. The combined system of the signal field and the atoms is in the state

$$\begin{aligned}|\Psi(0)\rangle &= \sum_{\{q_\mathrm{s}\}}\alpha(q_1,\cdots,q_\mathrm{s})|n_1\cdots n_\mathrm{s}\rangle|C^0\rangle \\ &= \sum_{\{q_\mathrm{s}\}}\alpha(q_1,\cdots,q_\mathrm{s})|D_{q_1}^{n_1}\cdots D_{q_\mathrm{s}}^{n_\mathrm{s}}\rangle_{\theta=0}. \end{aligned} \quad (64)$$

When the control field is strong enough, the signal pulse can maintain and transmit through the medium. Note that $|\Psi(0)\rangle$ is also the eigenstate of the interaction Hamiltonian with a null eigenvalue. If the control field is changed adiabatically to a very small level at a later time t_1, the state of the system is also changed adiabatically to

$$|\Psi(t_1)\rangle = \sum_{\{q_s\}} \alpha(q_1, \cdots, q_s)|D_{q_1}^{n_1} \cdots D_{q_s}^{n_s}\rangle_{\theta=\pi/2} \quad (65)$$
$$= \sum_{\{q_s\}} (-1)^{n_1+\cdots+n_s} \alpha(q_1, \cdots, q_s)|0\rangle|C_{q_1}^{n_1} \cdots C_{q_s}^{n_s}\rangle.$$

It forms an associate state for $|\Psi(0)\rangle$. The whole of the quantum information of the signal pulse has been stored in the atomic medium, in the form of a "negative copy", in which each excitation changes a π-phase. As soon as the control field returns to the previous level, the state (64) is recovered. Conversely, if equation (65) is an initial state generated in other model, by turning on the control field, it will be converted to the corresponding optical field, enabling it to be seen.

6 Dynamic quantum transfer in macroscopic matter

Due to the fact that the dark states are eigenstates of the EIT interaction, quantum transfer processes between field and matter are quasi-stationary by adiabatically changing the control field. On the other hand, the transfer can be performed in a dynamic way, which has been described in the literature [7,8]. In this section, we study a general description for dynamic transfer of quantum state between field and ultracold matter. The interaction configuration can be designed as, either, the parametric process or the Raman transition [7], in which both a weak signal beam and a strong control beam interact resonantly with two levels of atoms.

For simplicity, we consider a single-mode interaction. In the interaction picture, the effective interaction Hamiltonian is written as

$$H_{\mathrm{I}} = \hbar\Omega(a\sigma^\dagger + a^\dagger\sigma), \quad (66)$$

where Ω is the Rabi frequency of the control beam, assumed as classical. The collective atomic operator σ is defined by equation (3) and behaves boson-like in the low excitation limit. The model is well-known as a coupled harmonic oscillator, and can be solved exactly. Here we illustrate this model again from a new viewpoint by introducing a very simple method for the exact solution of the state-vector evolution. For this model, it is easy to obtain the evolution of the operators in the Heisenberg picture,

$$\begin{pmatrix} a(t) \\ \sigma(t) \end{pmatrix} = \begin{pmatrix} \cos\Omega t & -\mathrm{i}\sin\Omega t \\ -\mathrm{i}\sin\Omega t & \cos\Omega t \end{pmatrix} \begin{pmatrix} a(0) \\ \sigma(0) \end{pmatrix}. \quad (67)$$

With this method, if the initial state can be written as $|\Psi(0)\rangle = f(a(0), \sigma(0))|\Theta(0)\rangle$, while the evolution of the state $|\Theta(0)\rangle$ is already known to be $|\Theta(t)\rangle$, we obtain

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle = U(t)f(a(0), \sigma(0))|\Theta(0)\rangle$$
$$= U(t)f(a(0), \sigma(0))U^{-1}(t)U(t)|\Theta(0)\rangle$$
$$= f(a(-t), \sigma(-t))|\Theta(t)\rangle, \quad (68)$$

where $U(t) = \exp(-\mathrm{i}H_{\mathrm{I}}t/\hbar)$. An initial Fock state for the signal photons m and the atomic collective excitations n is represented by $|m, C^n\rangle = (1/\sqrt{m!n!})[a^\dagger(0)]^m[\sigma^\dagger(0)]^n|0, C^0\rangle$. Since the evolution of the vacuum state, as $|\Theta(0)\rangle$ in equation (68), is known to be $|\Theta(t)\rangle = \exp(-\mathrm{i}H_{\mathrm{I}}t/\hbar)|0, C^0\rangle = |0, C^0\rangle$, we obtain the evolution for an initial Fock state $|m, C^n\rangle$

$$|\Psi_{mn}(t)\rangle = (1/\sqrt{m!n!})[a^\dagger(-t)]^m[\sigma^\dagger(-t)]^n|0, C^0\rangle, \quad (69)$$

where the dynamical operators are written as

$$a^\dagger(-t) = U(t)a^\dagger U^{-1}(t) = a^\dagger\cos\Omega t - \mathrm{i}\sigma^\dagger\sin\Omega t, \quad (70a)$$
$$\sigma^\dagger(-t) = U(t)\sigma^\dagger U^{-1}(t) = \sigma^\dagger\cos\Omega t - \mathrm{i}a^\dagger\sin\Omega t, \quad (70b)$$

where $a^\dagger \equiv a^\dagger(0)$ and $\sigma^\dagger \equiv \sigma^\dagger(0)$. According to equation (69), the exact evolution for an arbitrary initial state $\sum \xi_{mn}|m, C^n\rangle$ is therefore $\sum \xi_{mn}|\Psi_{mn}(t)\rangle$.

The state $|\Psi_{mn}(t)\rangle$ described by equation (69) evolves periodically with the fundamental frequency Ω and conserves the total particle number. For example, in the simplest case of only a single excitation $|1, C^0\rangle$, the time evolution is $|\Psi_{10}(t)\rangle = \cos\Omega t|1, C^0\rangle - \mathrm{i}\sin\Omega t|0, C^1\rangle$. The entanglement between two subsystems increases in time and reaches a maximum at $\Omega t = \pi/4$. Then, the entanglement decreases and the excitation transfers completely from one subsystem to another at $\Omega t = \pi/2$. In general, if the initial state has m signal photons and no excitation for atoms $|m, C^0\rangle$, the time evolution is written as

$$|\Psi_{m0}(t)\rangle = (1/\sqrt{m!})[a^\dagger(-t)]^m|0, C^0\rangle$$
$$= (1/\sqrt{m!})[a^\dagger\cos\Omega t - \mathrm{i}\sigma^\dagger\sin\Omega t]^m|0, C^0\rangle$$
$$= \sum_{j=0}^{m}(-\mathrm{i})^j\sqrt{\frac{m!}{j!(m-j)!}}(\cos\Omega t)^{m-j}$$
$$\times (\sin\Omega t)^j|m-j, C^j\rangle. \quad (71)$$

The evolution states can be called "dynamic polaritons", since they are compatible with the "stationary polaritons" defined by equation (56). At the times $\Omega t = (1/2)\pi$, π and $(3/2)\pi$, the evolution state has been de-entangled to $(-\mathrm{i})^m|0, C^m\rangle$, $(-1)^m|m, C^0\rangle$, and $\mathrm{i}^m|0, C^m\rangle$, respectively. This means, at certain times, the entanglement formed in the dynamical process can be cancelled and the excitations are transferred completely from one subsystem to another. When an initial state is an arbitrary superposition of Fock states for the signal beam and the "vacuum" for the ultracold atoms, *i.e.* $|\Psi(0)\rangle = |\Phi, C^0\rangle$ where $|\Phi\rangle = \sum \alpha_m|m\rangle$, it evolves to the states $|0, \Phi^{(-\mathrm{i})}\rangle$, $|\Phi^{(-)}, C^0\rangle$, and $|0, \Phi^{(\mathrm{i})}\rangle$ at the times $\Omega t = (1/2)\pi$, π and $(3/2)\pi$, respectively, and comes back to the original at $\Omega t = 2\pi$. Here we define three states associated with $|\Phi\rangle = \sum \alpha_m |m\rangle$

$$|\Phi^{(\pm\mathrm{i})}\rangle = \sum (\pm\mathrm{i})^m \alpha_m |m\rangle, \quad |\Phi^{(-)}\rangle = \sum (-1)^m \alpha_m |m\rangle. \tag{72}$$

Obviously, these associate states have the same particle distribution, but with different phase shifts in amplitudes. As a matter of fact, the phase factor in the superposition can be observed macroscopically. For instance, if $|\Phi\rangle$ is a coherent state $|\alpha\rangle$, one obtains the coherent states again for the associate states with a particular phase shift, *i.e.* $|\Phi^{(\pm\mathrm{i})}\rangle = |\pm\mathrm{i}\alpha\rangle$ and $|\Phi^{(-)}\rangle = |-\alpha\rangle$. As for an arbitrary state, it is also true by means of the expectation value of the amplitude operator, such that $\langle\Phi^{(\pm\mathrm{i})}|a|\Phi^{(\pm\mathrm{i})}\rangle = \pm\mathrm{i}\langle\Phi|a|\Phi\rangle$ and $\langle\Phi^{(-)}|a|\Phi^{(-)}\rangle = -\langle\Phi|a|\Phi\rangle$. Though, in general, these associate states are not identical to the original, owing to a phase shift; the quantum information of the original state can still be faithfully preserved. It looks like a photograph and the corresponding negative copy. The associate states $|\Phi^{(\pm\mathrm{i})}\rangle$ and $|\Phi^{(-)}\rangle$ can be seen as "orthogonal" and "negative" copies of a quantum "picture" $|\Phi\rangle$.

In the dynamic quantum transfer, a quantum state of the signal field can be stored in and then retrieved from a medium by turning off and on the control field at a certain time. Because the dynamic polariton state defined by equation (71) is the eigenstate of the free Hamiltonian, it will be preserved while the interaction is turning off. Similarly, the model can be extended to the multimode case provided the conditions for the radiation wavelength equation (11) is satisfied. Thus, coherent information of the optical field can be transferred simultaneously.

Finally, we indicate that, for a proper transfer in this model, collective atoms must be prepared in an atomic storage state. For ultracold matter, its initial state can be considered as the vacuum state $|C^0\rangle$ approximately. If, at the initial time, the field is at an arbitrary state $|\Phi_1\rangle$ while collective atoms have been prepared in a superposition state $|\Phi_2\rangle = \sum \beta_m |C^m\rangle$, $|\Phi_1, \Phi_2\rangle$ will evolve to the corresponding states $|\Phi_2^{(-\mathrm{i})}, \Phi_1^{(-\mathrm{i})}\rangle$, $|\Phi_1^{(-)}, \Phi_2^{(-)}\rangle$ and $|\Phi_2^{(\mathrm{i})}, \Phi_1^{(\mathrm{i})}\rangle$ at the certain times mentioned above. It displays a complete swapping of quantum states for the coupled harmonic oscillators.

7 Conclusion

In conclusion, we define collective atomic operators and atomic storage states by containing spatial coherence and illustrate the conditions under which the multimode collective atomic lower and upper operators are boson-like. We indicate the fact that the atomic storage states shown by definition (16) are Dicke states with the maximum cooperation number. The new feature for these Dicke states is that, in the low excitation limit for a large number of atoms, they behave as the Fock states of an electromagnetic field. The complete description and the deductive explicit expressions for the atomic storage states present better physical understanding of why the atomic ensemble can record fully the quantum information, both the excitation and the coherence, of an optical electromagnetic field. In addition to adiabatic quantum transfer by means of dark states in EIT, we discuss the mechanism of dynamic quantum transfer via dynamic polaritons which is formed by the fundamental interaction between the field and ultracold matter. A combination of adiabatic and dynamic schemes may find more applications in quantum information technology.

This research was supported by the National Fundamental Research Project (973) No. 2001CB309310 and the National Natural Science Foundation of China, Project No. 10074008.

Appendix A

First, we derive the exact equation (23). By applying definitions (2) and (16), it gives

$$\begin{aligned}\sigma_k|C_k^n\rangle = \frac{1}{\sqrt{N}}\sum_{l=1}^{N}|b_l\rangle\langle c_l|\exp[-\mathrm{i}kz_l]\sqrt{\frac{n!}{N\cdots(N-n+1)}}\\ \times \sum_{\{i_n\}}{}'' |b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle\\ \times \exp[\mathrm{i}k(z_{i_1}+\cdots+z_{i_n})].\end{aligned} \tag{A.1}$$

When the operator $\sum_{l=1}^{N}|b_l\rangle\langle c_l|$ applies to a particular state $|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle$, it produces a superposition of n states in which $|c_{i_k}\rangle$ is sequentially replaced by $|b_{i_k}\rangle$. After this operation, equation (A.1) is a summation of $n \times \binom{N}{n} = N(N-1)\cdots(N-n+1)/(n-1)!$ states, in which $n-1$ atoms are populated in the level $|c\rangle$. It is in fact $N-n+1$ times $|C_k^{n-1}\rangle$. For example, a particular state in $|C_k^{n-1}\rangle$, say, $|c_1\cdots c_{n-1}b_n\cdots b_N\rangle$, comes from $N-(n-1)$ states, $|c_1\cdots c_{n-1}c_n b_{n+1}\cdots b_N\rangle$, $|c_1\cdots c_{n-1}b_n c_{n+1}\cdots b_N\rangle$, ..., $|c_1\cdots c_{n-1}b_n b_{n+1}\cdots c_N\rangle$, in $|C_k^n\rangle$. Thus, equation (A.1) is written as

$$\begin{aligned}\sigma_k|C_k^n\rangle &= \frac{1}{\sqrt{N}}\sqrt{\frac{n!}{N\cdots(N-n+1)}}(N-n+1)\\ &\quad\times \sum_{\{i_{n-1}\}}{}'' |b_1\cdots c_{i_1}\cdots c_{i_{n-1}}\cdots b_N\rangle\\ &\quad\times \exp[\mathrm{i}k(z_{i_1}+\cdots+z_{i_{n-1}})]\\ &= \frac{1}{\sqrt{N}}\sqrt{\frac{n!}{N\cdots(N-n+1)}}(N-n+1)\\ &\quad\times\sqrt{\frac{N\cdots(N-n+2)}{(n-1)!}}|C_k^{n-1}\rangle\\ &= \sqrt{\frac{N-n+1}{N}}\sqrt{n}|C_k^{n-1}\rangle\,.\end{aligned} \tag{A.2}$$

Then, we prove equations (27). Equation (27a) is written as

$$\sum_{l=1}^{N}(|b_l\rangle\langle b_l|C_k^n\rangle = \sqrt{\frac{n!}{N\cdots(N-n+1)}} \times \sum_{l=1}^{N}\sum_{\{i_n\}}{}''|b_l\rangle\langle b_l|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \times \exp[\mathrm{i}k(z_{i_1}+\cdots+z_{i_n})]$$
$$= \sqrt{\frac{n!}{N\cdots(N-n+1)}} \times \sum_{l=1}^{N}\sum_{\{i_n\}}{}''|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \times \exp[\mathrm{i}k(z_{i_1}+\cdots+z_{i_n})] \times (1-\delta_{li_1})\cdots(1-\delta_{li_n}). \quad (A.3)$$

Because all the indices of i_k are not equal to one another, one has

$$(1-\delta_{li_1})\cdots(1-\delta_{li_n}) = 1-(\delta_{li_1}+\cdots+\delta_{li_n}) + (\delta_{li_1}\delta_{li_2}+\cdots)-\cdots = 1-(\delta_{li_1}+\cdots+\delta_{li_n}). \quad (A.4)$$

Substituting equation (A.4) into equation (A.3), one obtains

$$\sum_{l=1}^{N}(|b_l\rangle\langle b_l|C_k^n\rangle = (N-n)|C_k^n\rangle\cdot \quad (A.5)$$

Equation (27b) is proved as

$$\sum_{l=1}^{N}(|c_l\rangle\langle c_l|C_k^n\rangle = \sqrt{\frac{n!}{N\cdots(N-n+1)}} \times \sum_{l=1}^{N}\sum_{\{i_n\}}{}''|c_l\rangle\langle c_l|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \times \exp[\mathrm{i}k(z_{i_1}+\cdots+z_{i_n})]$$
$$= \sqrt{\frac{n!}{N\cdots(N-n+1)}} \times \sum_{l=1}^{N}\sum_{\{i_n\}}{}''|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \times \exp[\mathrm{i}k(z_{i_1}+\cdots+z_{i_n})] \times (\delta_{li_1}+\cdots+\delta_{li_n})$$
$$= n|C_k^n\rangle\cdot \quad (A.6)$$

Appendix B

In this appendix, we calculate the normalized coefficient of the multimode storage states. A multimode single-excitation state is defined as

$$|C_{k_1}^1\cdots C_{k_n}^1\rangle \equiv \alpha\sum_{\{i_n\}}{}''|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \times \sum_{l=1}^{n!}\exp[\mathrm{i}\{k_n\}\cdot\{z_{i_n}\}_l]. \quad (B.1)$$

The probability of each state in the above superposition is

$$\left|\sum_{l=1}^{n!}\exp[\mathrm{i}\{k_n\}\cdot\{z_{i_n}\}_l]\right|^2 = \sum_{l,j}^{n!}\exp[\mathrm{i}\{k_n\}\cdot(\{z_{i_n}\}_l-\{z_{i_n}\}_j)]$$
$$= n! + \sum_{l\neq j}^{n!}\exp[\mathrm{i}\{k_n\}\cdot(\{z_{i_n}\}_l-\{z_{i_n}\}_j)]. \quad (B.2)$$

Then, we sum all these probabilities. For the first term of equation (B.2), it is simply

$$\sum_{\{i_n\}}{}''n! = n!\frac{N(N-1)\cdots(N-n+1)}{n!} = N(N-1)\cdots(N-n+1). \quad (B.3)$$

In the multimode case, one must find a mode with $k_j \neq 0$. By using equation (9), the summation $\sum''_{\{i_n\}}$ to the second term of equation (B.2) vanishes. Therefore, we obtain

$$\alpha = \frac{1}{\sqrt{N(N-1)\cdots(N-n+1)}}\cdot \quad (B.4)$$

Similarly, for a general multimode storage state defined by equation (40), the probability of finding a single state is

$$\left|\sum_{l=1}^{n!/(m_1!\cdots m_\mathrm{s}!)}\exp[\mathrm{i}\{k_\mathrm{s}^{(m_\mathrm{s})}\}\cdot\{z_{i_n}\}_l]\right|^2 = \sum_{l,j}^{n!/(m_1!\cdots m_\mathrm{s}!)}\exp[\mathrm{i}\{k_\mathrm{s}^{(m_\mathrm{s})}\}\cdot(\{z_{i_n}\}_l-\{z_{i_n}\}_j)]$$
$$= \frac{n!}{m_1!\cdots m_\mathrm{s}!} + \sum_{l\neq j}^{n!/(m_1!\cdots m_\mathrm{s}!)}\exp\left[\mathrm{i}\{k_\mathrm{s}^{(m_\mathrm{s})}\} \times (\{z_{i_n}\}_l-\{z_{i_n}\}_j)\right]. \quad (B.5)$$

The summation to the first term of the above equation gives

$$\sum_{\{i_n\}}{}''\frac{n!}{m_1!\cdots m_\mathrm{s}!} = \frac{n!}{m_1!\cdots m_\mathrm{s}!}\frac{N(N-1)\cdots(N-n+1)}{n!} = \frac{N(N-1)\cdots(N-n+1)}{m_1!\cdots m_\mathrm{s}!}\cdot \quad (B.6)$$

With the same reason, the summation to the second term vanishes. The normalized coefficient is therefore

$$\alpha = \sqrt{\frac{m_1!\cdots m_\mathrm{s}!}{N(N-1)\cdots(N-n+1)}}\cdot \quad (B.7)$$

Appendix C

We define a new collective atomic state in which n atoms are in the level $|c\rangle$ whereas one atom is in the level $|a\rangle$

$$|A_q^1, C_q^n\rangle \equiv \sqrt{\frac{n!}{N(N-1)\cdots(N-n)}} \times \sum_{l\neq\{i_n\}} \sum_{\{i_n\}}{}'' |b_1\cdots a_l \cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \exp[\mathrm{i}(k_\mathrm{s}+q-k_\mathrm{c})(z_{i_1}+\cdots+z_{i_n})]\exp[\mathrm{i}(k_\mathrm{s}+q)z_l], \quad \text{(C.1)}$$

where $\sum_{l\neq\{i_n\}}$ designates the summation for index l which cannot be taken as $i_1, ..., i_n$. We have already indicated that, the state $|C_q^n\rangle$ is a superposition of $N(N-1)\cdots(N-n+1)/n!$ possible states $|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle$ in which n atoms are in the level $|c\rangle$ whereas the remaining $N-n$ atoms are in the level $|b\rangle$. For one of these states, each of the $N-n$ atoms being in the level $|b\rangle$ can be excited to the level $|a\rangle$. So the state $|A_q^1, C_q^n\rangle$ includes $N(N-1)\cdots(N-n)/n!$ such possible states $|b_1\cdots a_l\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle$ with equal possibility. The state $|A_q^1, C_q^n\rangle$ has been normalized. The phase factor related to the excited atom l being in level $|a\rangle$ is $\exp[\mathrm{i}(k_\mathrm{s}+q)z_l]$, because the transition of the level $|a\rangle$ to the ground level $|b\rangle$ is connected with the signal field of the wavevector $k_\mathrm{s}+q$. State (C.1) can be obtained by the following operation

$$\begin{aligned} N\rho_\mathrm{ac}(0)|C_q^n\rangle &= \left(\sum_{l=1}^N |a_l\rangle\langle c_l|\exp[\mathrm{i}k_\mathrm{c}z_l]\right)|C_q^n\rangle \\ &= \sqrt{\frac{n!}{N\cdots(N-n+1)}} \sum_{l=1}^N \sum_{\{i_n\}}{}'' |a_l\rangle\langle c_l|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \exp[\mathrm{i}(k_\mathrm{s}+q-k_\mathrm{c})(z_{i_1}+\cdots+z_{i_n})]\exp[\mathrm{i}k_\mathrm{c}z_l] \\ &= \sqrt{\frac{n!}{N\cdots(N-n+1)}} \sum_{l\neq\{i_{n-1}\}} \sum_{\{i_{n-1}\}}{}'' |b_1\cdots a_l\cdots c_{i_1}\cdots c_{i_{n-1}}\cdots b_N\rangle \\ &\qquad \times \exp[\mathrm{i}(k_\mathrm{s}+q-k_\mathrm{c})(z_{i_1}+\cdots+z_{i_{n-1}})]\exp[\mathrm{i}(k_\mathrm{s}+q)z_l] \\ &= \sqrt{\frac{n!}{N\cdots(N-n+1)}}\sqrt{\frac{N\cdots(N-n+1)}{(n-1)!}}|A_q^1, C_q^{n-1}\rangle = \sqrt{n}|A_q^1, C_q^{n-1}\rangle, \end{aligned} \quad \text{(C.2)}$$

where the atomic operator $\rho_\mathrm{ac}(0)$ has been defined in equation (48b). Similarly, we have

$$\begin{aligned} N\rho_\mathrm{ab}(q)|C_q^n\rangle &= \left(\sum_{l=1}^N |a_l\rangle\langle b_l|\exp[\mathrm{i}(k_\mathrm{s}+q)z_l]\right)|C_q^n\rangle \\ &= \sqrt{\frac{n!}{N\cdots(N-n+1)}} \sum_{l=1}^N \sum_{\{i_n\}}{}'' |a_l\rangle\langle b_l|b_1\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \exp[\mathrm{i}(k_\mathrm{s}+q-k_\mathrm{c})(z_{i_1}+\cdots+z_{i_n})]\exp[\mathrm{i}(k_\mathrm{s}+q)z_l] \\ &= \sqrt{\frac{n!}{N\cdots(N-n+1)}} \sum_{l\neq\{i_n\}} \sum_{\{i_n\}}{}'' |b_1\cdots a_l\cdots c_{i_1}\cdots c_{i_n}\cdots b_N\rangle \exp[\mathrm{i}(k_\mathrm{s}+q-k_\mathrm{c})(z_{i_1}+\cdots+z_{i_n})]\exp[\mathrm{i}(k_\mathrm{s}+q)z_l] \\ &= \sqrt{\frac{n!}{N\cdots(N-n+1)}}\sqrt{\frac{N\cdots(N-n)}{n!}}|A_q^1, C_q^n\rangle = \sqrt{N-n}|A_q^1, C_q^n\rangle \cdot \end{aligned} \quad \text{(C.3)}$$

The two interactions induced by two fields in the interaction Hamiltonian interfere destructively for the dark state. Using equations (C.2, C.3), for the exact expression of the dark state equation (55), one obtains

$$\begin{aligned} \Omega N\rho_\mathrm{ac}(0)|D_q^n\rangle &= \Omega N\rho_\mathrm{ac}(0)\sum_{m=0}^n (-1)^m \sqrt{\frac{n(n-1)\cdots(n-m+1)}{m!}}\sqrt{\frac{N(N-1)\cdots(N-m+1)}{N^m}} \\ &\qquad \times \frac{\Omega^{n-m}(g\sqrt{N})^m}{(\Omega^2+g^2N)^{n/2}}|n-m\rangle|C_q^m\rangle \\ &= \sum_{m=1}^n (-1)^m \sqrt{\frac{n(n-1)\cdots(n-m+1)}{(m-1)!}}\sqrt{\frac{N(N-1)\cdots(N-m+1)}{N^m}}\frac{\Omega^{n-m+1}(g\sqrt{N})^m}{(\Omega^2+g^2N)^{n/2}}|n-m\rangle|A_q^1, C_q^{m-1}\rangle \end{aligned} \quad \text{(C.4)}$$

and

$$gNa(q)\rho_{\rm ab}(q)|D_q^n\rangle = gNa(q)\rho_{\rm ab}(q)\sum_{m=0}^{n}(-1)^m\sqrt{\frac{n(n-1)\cdots(n-m+1)}{m!}}\sqrt{\frac{N(N-1)\cdots(N-m+1)}{N^m}}$$
$$\times\frac{\Omega^{n-m}(g\sqrt{N})^m}{(\Omega^2+g^2N)^{n/2}}|n-m\rangle|C_q^m\rangle$$
$$=\sum_{m=0}^{n-1}(-1)^m\sqrt{\frac{n(n-1)\cdots(n-m)}{m!}}\sqrt{\frac{N(N-1)\cdots(N-m)}{N^{m+1}}}\frac{\Omega^{n-m}(g\sqrt{N})^{m+1}}{(\Omega^2+g^2N)^{n/2}}|n-m-1\rangle|A_q^1,C_q^m\rangle\cdot \quad \text{(C.5)}$$

If we set index $m \to m+1$ in equation (C.4), it is the exact same as equation (C.5) but with an opposite sign. Therefore, one obtains

$$[gNa(q)\rho_{\rm ab}(q)+\Omega N\rho_{\rm ac}(0)]|D_q^n\rangle = 0. \quad \text{(C.6)}$$

Resulting in, for the exact resonant mode $q=0$, the dark states $|D_{q=0}^n\rangle$ being the eigenstates with the null eigenvalue of the interaction Hamiltonian (49). We note that equations (C.4–C.6) hold exactly for the exact expression of the dark state (55). For the approximate expression of the dark state (56), equations (C.4–C.6) are also satisfied as long $N \gg n$.

References

1. C. Liu, Z. Dutton, C.H. Behroozi, L.V. Hau, Nature **409**, 490 (2001)
2. D.F. Phillips, A. Fleischhauer, A. Mair, R.L. Walsworth, Phys. Rev. Lett. **86**, 783 (2001)
3. M. Fleischhauer, M.D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000)
4. M.D. Lukin, S.F. Yelin, M. Fleischhauer, Phys. Rev. Lett. **84**, 4232 (2000)
5. M.D. Lukin, A. Imamoglu, Nature **413**, 273 (2001)
6. O. Kocharovskaya, Y. Rostovtsev, M.O. Scully, Phys. Rev. Lett. **86**, 628 (2001)
7. A.E. Kozhekin, K. Mølmer, E. Polzik, Phys. Rev. A **62**, 033809 (2000)
8. A.B. Matsko, Y.V. Rostovtsev, O. Kocharovskaya, A.S. Zibrov, M.O. Scully, Phys. Rev. A **64**, 043809 (2001)
9. M. Fleischhauer, M.D. Lukin, Phys. Rev. A **65**, 022314 (2002)
10. S.E. Harris, Phys. Today **50**, 36 (1997)
11. E. Arimondo, Prog. Opt. **35**, 259 (1996)
12. L.V. Hau, S.E. Harris, Z. Dutton, C.H. Behroozi, Nature **397**, 594 (1999)
13. M.M. Kash, V.A. Sautenkov, A.S. Zibrov, L. Hollberg, G.R. Welch, M.D. Lukin, Y. Rostovtsev, E.S. Fry, M.O. Scully, Phys. Rev. Lett. **82**, 5229 (1999)
14. D. Budker, D.F. Kimball, S.M. Rochester, V.V. Yashchuk, Phys. Rev. Lett. **83**, 1767 (1999)
15. R.H. Dicke, Phys. Rev. **93**, 99 (1954)
16. Wang Kai-Ge, Zhu Shi-Yao, Chin. Phys. Lett. **19**, 56 (2002)